

Dislocations and Nonelastic Processes in the Mantle

J. BERNARD MINSTER AND DON L. ANDERSON

Seismological Laboratory, California Institute of Technology, Pasadena, California 91125

Dislocations in solids contribute to anelastic attenuation, relaxation of the shear modulus, transient creep, and steady state flow. These properties of the mantle may therefore be related. The glide and climb of dislocations appear to have the appropriate time constants to explain seismic wave attenuation and mantle viscosity, respectively. The dislocation density of the mantle depends on the ambient stress. The characteristic time scales of dislocation relaxation depend on dislocation length and temperature. These time scales for the mantle can be inferred from seismic wave attenuation and postglacial rebound, thereby potentially yielding information about dislocation density, stress, and temperature. The thickness of the 'rheological' lithosphere depends on stress and duration of load as well as age. Kilobar level stresses can be supported in the lithosphere for times greater than 10^6 years. The relaxation time decreases rapidly with temperature. The asthenosphere can therefore only support small stresses on time scales of geological interest.

INTRODUCTION

Dislocations contribute to steady state creep via multiplication and annihilation and to anelastic attenuation via reversible stress-induced bowing. Internal friction and creep experiments are both commonly used to elucidate dislocation structure and dynamics in metals and other materials of industrial significance, but the possibility that anelasticity at seismic and ultrasonic frequencies might be closely related to longer term processes, such as creep in the mantle, has not been much explored in geophysics. There is now enough physical property data on one mantle silicate, olivine, to calculate the creep rate as a function of stress and temperature from such parameters as the diffusivity, dislocation density, subgrain size, etc. and to compare the theoretical predictions with observed laboratory creep data. It is also possible to estimate the short-time, low-stress behavior of a dislocated solid and to compare this with seismic and laboratory attenuation data.

We conclude that dynamic testing of stressed silicates would be a valuable complement to conventional creep experiments, particularly for the understanding of dislocation physics. We also demonstrate the plausibility of a relation between mantle viscosity and seismic wave attenuation via the stress, which controls the dislocation microstructure.

CHARACTERISTIC TIMES

The most convenient comparison of relaxation and creep mechanisms is achieved by recasting theoretical results and experimental data in terms of characteristic times τ . For creep processes this is simply the Maxwell time, the ratio of viscosity η to rigidity G :

$$\tau = \eta/G = (\sigma/\dot{\epsilon}) \quad (1)$$

where σ is the deviatoric stress and $\dot{\epsilon}$ the strain rate. For the upper mantle, τ can be calculated from postglacial rebound data [e.g., *Cathles, 1975*] to be 10^9 – 10^{10} s. This parameter can also be derived from laboratory creep data and from theories of creep. For dislocation creep models we use the simple form of Orowan's equation discussed by *Poirier [1976]*:

$$\dot{\epsilon} = \beta \rho_m b V_c \quad (2)$$

where ρ_m is the area density of mobile dislocations, b is Burgers' vector, and β is the ratio of the mean free path of the dislocation to the distance covered at the rate controlling speed V_c (usually the climb velocity). Then

$$\tau = \sigma/G\beta\rho_m b V_c \quad (3a)$$

For anelastic relaxation due to diffusion-controlled bowing of a dislocation link of length l strongly pinned at both ends the relaxation time can be written [*Schoeck, 1963; Simpson and Sosin, 1972; Minster and Anderson, 1980*]

$$\tau = \sigma l^2/G\pi^2 b V_D \quad (3b)$$

where V_D is the steady state drift velocity (glide and/or climb) of a straight dislocation under the applied stress σ . Seismic waves with periods of 10–100 s are strongly attenuated in the upper mantle. This suggests a short-term relaxation time of this order. The low attenuation in the lithosphere and lower mantle indicates that the characteristic relaxation times in these regions are well removed from the seismic band. This is presumably a result of low temperatures in the former case and high pressures in the latter, both of which serve to increase the relaxation time.

A general form for the velocity of diffusion-controlled motion of a dislocation under applied stress σ is

$$V \propto \sigma b^2 D/kT$$

where the diffusivity D pertains to the rate-controlling process. For thermally activated processes,

$$\tau = \tau_0 \exp(E^*/RT) \quad (4a)$$

and we have

$$\tau_0 = AkT\lambda^2/D_0 b^3 G \quad (4b)$$

Here A is a dimensionless, model-dependent constant, D_0 is the appropriate diffusion constant, and λ is a scale length. For models of dislocation relaxation, which usually involve noninteracting dislocation links, we have $\lambda = l$, the link length between strong pinners. For steady creep, λ is more conveniently chosen to be $\rho_m^{-1/2}$. Of course, for a Frank network we have $\rho_m l^2 = \rho l^2 = 1$, and using the scaling relation,

$$\rho b^2 = (\sigma/G)^2$$

We obtain the classical dislocation creep equation

$$\dot{\epsilon} = (D_0 G b / A k T) (\sigma / G)^3 \exp(-E^* / RT) \quad (5)$$

For example, in Nabarro's [1967] model of creep controlled by dislocation climb in a uniform network, $A = \pi \ln(4G/\pi\sigma)$, and the activation energy is that for diffusion of the slowest-moving species.

Equation (4) remains valid if creep is controlled by kink or jog formation [e.g., Hirth and Lothe, 1968; Gittus, 1976] or by the diffusion of point defects bound to the dislocation [e.g., Schoeck, 1963]. In climb theories the activation energy is that for self-diffusion plus a contribution from the jog formation energy if we have jog undersaturation. In glide theories the activation energy involves the kink formation energy and/or the effective activation energy (i.e., corrected for the binding energy) for diffusion of point defects bound to the dislocation line [DeBatist, 1972]. The activation energy for climb is generally much greater than that for glide, although jog drag-controlled glide is an obvious exception [e.g., Hirth and Lothe, 1968].

When dislocation glide is rate limited by the diffusion of impurity atoms bound to the dislocation line, then A is proportional to C_L , the atomic fraction of impurities in the lattice, and E^* is the sum of the impurity diffusion activation energy and the binding energy [Miner et al., 1976].

In the simple model of glide limited by Peierls barriers presented by Hirth and Lothe [1968] we have

$$A \propto b^2 G / 2 \lambda^2 E_k$$

where E_k is the kink formation energy. The effective activation energy is then $2E_k$ and $D_0 = D_k$, the kink diffusion constant.

In all cases the actual value of τ depends rather strongly on the geometry of the microstructure.

CREEP RATE

In many solids undergoing slow deformation and, in particular, olivine [Durham et al., 1977], subgrain formation is observed, and dislocations tend to concentrate in cell walls. If creep is controlled by recovery in the cell walls, then the parameter β in (3a) and thus the constant A depend on the ratio of cell diameter to the mean dislocation spacing [e.g., Weertman, 1975; Gittus, 1976]. The longest dislocations are then bounded by the grain size $L = K_s G b / \sigma$, where K_s is about 15 for olivine.

In that case, A is of the general form

$$A^{-1} = \gamma K_s^m \quad (6)$$

where γ and m are constants which depend on the details of the mechanism.

For example, if all the dislocations are in the boundaries, these subgrains undergo Nabarro-Herring type creep, and $\gamma \sim 5$, $m = -2$ [Weertman, 1975]. Models involving dislocation annihilation in cell walls lead to the same form of the equations, but the constants γ and m depend on whether climb in the cell walls takes place under external stress or self stress and on the distance over which climb takes place before annihilation. In the model of Ivanov and Yanushkevich [1964], annihilation of dislocations of opposite signs, originating from adjacent subgrains, takes place in the cell wall by climb over $H/4$, where H is the mean separation of intracell dislocations. In that case, $\gamma \sim 0.2$, $m = 0$, and the cell size is eliminated from the creep

rate. In contrast, Gittus [1976] assumed that most dislocations are to be found within cell walls and that recovery is controlled by climb in the walls under self stress, so as to maintain a constant dislocation density. He finds $A^{-1} = K^6$, where $K = L/\sqrt{\rho}$ and ρ is the smeared dislocation density. Using the observed relation for olivine [Durham et al., 1977], $\rho_m^{-1/2} = 1.66 G b / \sigma$; this leads to $m = 6$ and $\gamma \sim 0.05 (\rho/\rho_m)^3$. If ρ is an order of magnitude greater than ρ_m , $\gamma \sim 50$. In the dislocation pile-up model of Weertman [1975], equation (6) still holds with $m = 3$, $\gamma = 0.006$.

Various models have been proposed in the literature which lead to the appearance of the ratio σ/G in γ and thus result in a different power law dependence in (5), but as pointed out by Weertman [1975] and Poirier [1976], such models usually involve ad hoc modifications, and the σ^3 dependence is the natural one.

A noteworthy feature of cell wall recovery models is that for high dislocations densities the condition for jog saturation may not be satisfied. The effective activation energy in this case is $E^* = E_{SD} + E_j$, where E_{SD} and E_j are the self-diffusion and jog formation energies, respectively. For creep of olivine the apparent activation energy is 125 kcal/mol [Kohlstedt and Goetze, 1974; Goetze, 1978]. The diffusion activation energies for Si^{3+} and O^{2-} in olivine are both about 90 kcal/mol [Reddy et al., 1980; Poumellec et al., 1980]. The discrepancy could be attributed to jog formation [Gueguen, 1979], implying $E_j \sim 35$ kcal/mol. In contrast, E_j is usually much smaller for metals, so that $E^* \sim E_{SD}$.

Poumellec et al. [1980] have recently proposed that creep in olivine is rate limited by the diffusion of Si. With their estimate, $D_0(\text{Si}) \sim 1.5 \cdot 10^{-6} \text{ cm}^2/\text{s}$, (5) is plotted in Figure 1 with the olivine creep data of Kohlstedt et al. [1976]. If silicon is the controlling diffusing species, then K from the theory of Gittus [1976] is about 25–35, so that $K_s \sim 12$ –17, in agreement with observations. If oxygen is the controlling species, then with D_0 (oxygen) $\sim 3.5 \cdot 10^{-3} \text{ cm}^2/\text{s}$ [Reddy et al., 1980] we get $K \sim 7$ to 10, or $K_s \sim 3$ to 5. The other parameters in (5) have been taken from Ashby and Verrall [1978].

Obviously, there is a trade-off between γ , K_s , and D_0 , and we can equally well consider $\gamma K_s^m D_0$ to be an empirical parameter with a value of $8.9 \times 10^2 \text{ cm}^2/\text{s}$ determined by the creep data.

The characteristic times for climb in olivine with the above values are given in Table 1 for a range of stresses and temperatures that are appropriate for the upper mantle. Note that these agree with rebound data, $\tau \sim 10^{10} \text{ s}$, for combinations of temperature and stress ranging from 1700 K and 1 bar to 1500 K and 100 bars. If the temperature of the mantle is known independently, then the characteristic time associated with post-glacial uplift or postearthquake rebound can be used to estimate stress in the mantle. From magma temperatures and inclusions in xenoliths the temperature of the upper mantle in the vicinity of the low-velocity zone is about 1600 K. This is consistent with a stress level of a few bars. From the relation between τ and σ we can infer that a stress of a few kilobars in the asthenosphere would decay in 10^4 s .

RELAXATION TIME—THE SEISMIC PROBLEM

It is more difficult to estimate the characteristic time associated with dislocation damping in the upper mantle, since there is little pertinent experimental data. The idea that dislocations may be responsible for seismic wave attenuation is

based primarily on the behavior of deformed metals at high temperature [e.g., *Gueguen and Mercier, 1973*].

The stress-induced bowing of dislocations in the glide plane is a mechanism that has been proposed for high-temperature attenuation [e.g., *Hirth and Lothe, 1968; Schoeck, 1963*]. The characteristic time for bow-out of a single dislocation link of length l is given by (4) with $A = 2/\pi^2$. D_0 is approximately the square of the jump distance (~ 1 Å) times the Debye frequency ($\sim 10^{13}$ /s) or $\sim 10^{-3}$ cm²/s. This is about the same as the D_0 calculated for oxygen and magnesium-iron diffusion in olivine [*Reddy et al., 1980; Misener, 1974*]. This gives a τ_0 of 4×10^{-7} s for a stress of 10 bars and olivine parameters from *Stocker and Ashby [1973]*. With this estimate the relaxation time is brought into the seismic band at upper mantle temperatures if the controlling activation energy is of order 50–60 kcal/mol.

Dislocation glide is rate-limited by kink nucleation or by diffusion of point defects which are bound to the dislocation line. The controlling activation energy is generally much smaller than for steady state creep. For example, the activation energy for double-kink nucleation in olivine has been estimated to be about 52 kcal/mol [*Stocker and Ashby, 1973*]. The diffusion of point defects in olivine gives activation energies ranging from 50 to 90 kcal/mol. If core diffusion rather than lattice diffusion is important, the effective activation energy may be as small as one half the above values [e.g., *Hirth and Lothe, 1968*]. The only attenuation measurement on olivine at seismic stress levels and strain rates yields an activation energy of 57 kcal/mol for a high-temperature internal friction peak and 45 kcal/mol for the high-temperature background [*Jackson, 1969*].

The preexponential for dislocation glide is also different from that for climb. The effective dislocation length may be shorter if jogs act as pinners at seismic frequencies. For long-duration, large-strain creep experiments the activation energies and dislocation lengths for climb and glide may appear to be the same, since self-diffusion is involved in both the climb of dislocations and the glide of jogged dislocations. For rapid small-strain experiments the jogs do not have time to diffuse, and the time constant is controlled by the glide rate of dislocation segments between pinning points or by the nucleation rate of kinks if this is the slower process.

For purposes of illustration we assume that the activation

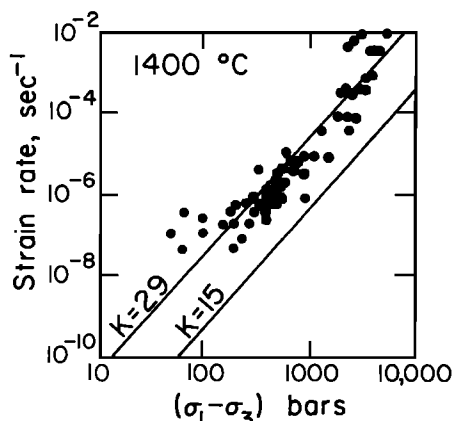


Fig. 1. Calculated strain rates versus deviatoric stress for olivine with the values for the K parameter (subgrain diameter/total dislocation density). Data are from *Kohlstedt et al. [1976]*. Olivine parameters are from *Ashby and Verrall [1978]* and *Poumellec et al. [1980]*.

TABLE 1. Relaxation Times for Glide and Climb as a Function of Tectonic Stress σ and Temperature T

	T, K		
	1400°	1600°	1800°
$\sigma = 1 \text{ bar}$			
Climb	10^{13}	10^{11}	10^9
Glide	10^4	10^3	10^2
$\sigma = 10 \text{ bars}$			
Climb	10^{11}	10^9	10^7
Glide	10^2	10^1	10^0
$\sigma = 100 \text{ bars}$			
Climb	10^9	10^7	10^5
Glide	10^0	10^{-1}	10^{-2}

energy for glide is 54 kcal/mol. This is in the range of values determined from attenuation measurements on olivine [*Jackson, 1969*]. It is also a reasonable value for the activation energy of diffusion of cations in silicates [*Ahrens and Schubert, 1975*]. Coincidentally, this is also about the value for double-kink nucleation using estimates of *Stocker and Ashby [1973]* for the kink-formation energy in olivine. For D_0 we take 10^{-3} cm²/s and assume that the dislocation length is given by $\rho_m^{-1/2} = Gb/\sigma$, where σ is the tectonic stress. The relaxation times for glide with these assumptions are given in Table 1. If there are pinning points along the dislocation line which are immobile on the time scale of a seismic wave, the effective dislocation length is decreased and the relaxation time is shortened. Although the numbers in Table 1 are uncertain, it appears plausible that dislocation glide may have the appropriate relaxation time to explain seismic wave attenuation in the upper mantle.

Mobile point defects also reduce τ . In *Schoeck's [1963]* theory the constant A in (4) is C_i ; the atomic fraction of mobile point defects along the dislocation line ($C_i < 1$). If $C_i = 10^{-3}$, for example, the values for glide in Table 1 would be reduced by three orders of magnitude. This shifts the peak frequency, but the absorption band remains in the seismic band for low values of tectonic stress.

We have discussed in a general way the factors which may influence the stress-induced glide of dislocations in crystals. The time dependence of the material response to an external load may be due to the discrete nature of the lattice (Peierls barrier) or to the interaction of the dislocations with other defects or impurities. The former is responsible for damping peaks found in relatively pure deformed metals at low temperature and high frequency. The latter is responsible for high-temperature, low-frequency peaks on a variety of impure metals [*Nowick and Berry, 1972; DeBatist, 1972*]. The role of 'dragging points' on dislocations has also been studied for crystals having high Peierls stress [*Celli et al., 1963*] and ionic crystals [*Menezes and Nix, 1974*], but the experimental base is very limited. This mechanism seems to apply to dislocated Al_2O_3 at high temperature [*Huber et al., 1961*]. Mantle silicates may contain impurities such as Fe^{3+} , Al^{3+} , etc. and also high concentrations of point defects such as Mg and Si interstitials and vacancies [*Stocker, 1978*]. The concentrations of these around dislocations and their effect on glide is unstudied. A number of high-temperature absorption peaks have been found in deformed ionic and covalent crystals, some of which apparently represent the bowing out of dislocations interacting with point defects that diffuse with the dislocations [*Nowick and Berry, 1972*].

The kink of formation energy is theoretically smaller than the jog formation energy [Hirth and Lothe, 1968], which we have estimated to be about 35 kcal/mol from creep data. The double-kink formation model for glide will therefore be controlled by an activation energy of less than 70 kcal/mol. Most cations in silicates have diffusional activation energies in the range 40 to 90 kcal/mol. It seems fairly certain that the activation energy for dislocation bowing in mantle silicates, whatever the mechanism, will be less than that for creep. This is a necessary condition if the same population of dislocations that controls creep in the mantle also contributes to damping at seismic frequencies.

As can be seen from Table 1, the difference in activation energies for climb and glide leads to very large differences in the two relaxation times. The shorter time constant, for a reasonable choice of parameters, is in the seismic band in the upper mantle. This region would therefore be characterized by high attenuation of seismic waves and a modulus lower than the high-frequency or low-temperature modulus. For long-period seismic waves the modulus is relaxed by glide, and we have a low-velocity zone.

For times shorter than τ_c , the climb time constant, the irreversible strain associated with self-diffusional processes is small. The characteristic time decreases with stress and temperature. This provides a working definition of the rheological or elastic lithosphere. For times less than τ_c the material is essentially anelastic; i.e., strains are time dependent but reversible. The thickness of the 'elastic' lithosphere is therefore a function of time and load as well as temperature.

Note that the characteristic relaxation times for dislocation damping depend on the square of the effective link length and thus on the square of the long-term steady state tectonic stress. This opens up the possibility that stress in the mantle or, at least, variations in stress from place to place can be estimated from both lithospheric loading experiments and from seismic wave attenuation. Obviously, there is need for low-strain laboratory experiments at seismic periods on prestressed silicates under a variety of conditions in order to exploit fully seismic methods for estimating stress and temperature in the mantle.

EFFECT OF A SPECTRUM OF RELAXATION TIMES

Relaxation mechanisms lead to an internal friction peak of the form

$$Q^{-1}(\omega) = \Delta \int_{-\infty}^{\infty} \mathcal{D}(\tau) \{ \omega\tau / (1 + \omega^2\tau^2) \} d\tau \quad (7)$$

where ω is the applied frequency, τ is a characteristic time, $\mathcal{D}(\tau)$ is the retardation spectrum, and Δ is the modulus defect, $(G_u - G_r)/G_r$, the relative difference between the high-frequency, unrelaxed shear modulus G_u , and the low-frequency, relaxed modulus G_r . The modulus defect is also a measure of the total reduction in modulus that is obtained in going from low temperature to high temperature. For a dislocation model the modulus defect is due to the strain contributed by dislocation bowing. The dislocations bow to an equilibrium radius of curvature which is dictated by the applied stress. The rate at which they do so is controlled by the propagation of kinks or diffusion of point defects near the dislocation and therefore is an exponential function of temperature.

A suitable form of the retardation spectrum is [Minster and Anderson, 1980]

$$\mathcal{D}(\tau) = [\alpha / (\tau_2^\alpha - \tau_1^\alpha)] \tau^{\alpha-1} \quad (8)$$

for $\tau_1 < \tau < \tau_2$ and zero elsewhere. τ_1 and τ_2 are the shortest and longest relaxation times respectively, of the mechanism being considered.

When $\tau_1 \sim \tau_2$, (7) reduces to the well-known Debye peak. When $\alpha = 0$ and $\tau_2 \gg \tau \gg \tau_1$, a frequency-independent Q results [Liu et al., 1976]. The more general case of $0 < \alpha < 1$ has recently been discussed [Anderson and Minster, 1979]. This leads to a weak dependence, $Q(\omega) \sim \omega^\alpha$ for $\tau_1 < \tau < \tau_2$ and has characteristics similar to Jeffreys' [1958] modification of Lomnitz' law. Outside the absorption band, $Q(\omega) \sim \omega^{\pm 1}$ for $\omega\tau \gg 1$; this result holds for all relaxation mechanisms.

The Q corresponding to (1) and (2) for frequencies ω such that

$$\omega\tau_1 \ll 1 \ll \omega\tau_2$$

is given by [Anderson and Minster, 1979; Macdonald, 1961]:

$$Q(\omega) = \cot \frac{\alpha\pi}{2} + \frac{2J_u}{\pi\alpha\delta J} [(\omega\tau_2)^\alpha - (\omega\tau_1)^\alpha] \cos \frac{\alpha\pi}{2} \\ \approx \frac{2J_u}{\pi\alpha\delta J} (\omega\tau_2)^\alpha \cos \frac{\alpha\pi}{2} \quad (9)$$

since $\tau_2 \gg \tau_1$ and $\cot \alpha\pi/2$ is small. J_u , J_r , and $\delta J = J_r - J_u$ are the unrelaxed, relaxed, and defect compliances, respectively.

For high frequencies, $1 \ll \omega\tau_1 \ll \omega\tau_2$

$$Q(\omega) = (1 - \alpha)(J_u/\alpha\delta J)(\tau_2/\tau_1)^\alpha (\omega\tau_1) \quad (10)$$

At low frequencies,

$$Q(\omega) = (1 + \alpha)(J_r/\alpha\delta J)(\omega\tau_2)^{-1} \quad (11)$$

For a thermally activated process the characteristic times depend on temperature according to (3). It is clear that at very low temperature the characteristic times are long and $Q \sim \omega$. At very high temperature, Q is predicted to increase as ω^{-1} . Equation (9) holds for intermediate temperatures.

The dispersion appropriate for the frequency-dependent Q model considered above gives, in the case $Q \gg 1$,

$$G_1/G_2 = 1 - \cot(\alpha\pi/2)(\omega_2/\omega_1)Q_2^{-1} \quad (12)$$

where Q_2 is the Q at ω_2 . The rigidities at the two frequencies are $G_1(\omega_1)$ and $G_2(\omega_2)$. This applies for $\omega\tau < \omega\tau_2$. For low frequencies, $G = G_r$. For high frequencies, $G = G_u$. For glide and a Frank network the difference between the relaxed and unrelaxed moduli is about 8%.

LOW-VELOCITY ZONE

The shear velocity in the mantle decreases by about 10% between 40 and 150 km in tectonic and oceanic regions [Anderson and Hart, 1976; Cara, 1979]. When corrected for temperature and pressure by using single crystal values for olivine [Kumazawa and Anderson, 1969], the remaining shear modulus defect is about 8%. This is the same as the modulus defect associated with dislocation glide [Minster and Anderson, 1980]. The total modulus defect associated with dislocations when both climb and glide are allowed is slightly greater. The climb component requires jog nucleation and self-diffusion and therefore only contributes at very high temperature, low frequency, or long time. By contrast, the decrease in velocity in the upper mantle under shields and nontectonic regions can be accounted for almost entirely by the high-frequency single crystal elastic constants and their pressure and temperature derivatives. A modulus defect of 8% gives a minimum Q at $\omega\tau = 1$ of 25, which is in good agreement with the seismic data

[Solomon, 1972]. Thus the reduction in velocity and the attenuation caused by dislocations bowing under small stresses are in excellent accord with the seismic data.

CREEP RESPONSE

The creep response of the standard linear solid with a single characteristic relaxation time (actually the strain retardation time) τ is

$$\epsilon(t) = \sigma[J_u + \delta J(1 - \exp(-t/\tau))] \quad (13)$$

where σ is the stress, J_u the unrelaxed compliance (the reciprocal of the conventional high-frequency elastic modulus), and δJ is the difference between the relaxed (i.e., long time or low frequency) and unrelaxed (i.e., short time or high frequency) compliances.

A more general model involves a spectrum of retardation times. For a distribution of characteristic times $D(\tau)$ the creep is

$$\epsilon(t) = \sigma \left\{ J_u + \sigma J \int_0^\infty \mathcal{A}(\tau) [1 - \exp(-t/\tau)] d\tau \right\} \quad (14)$$

The distribution function, equation (2), yields a creep response which can be approximated by

$$\epsilon(t) \approx \sigma J_u \left[1 + \frac{\delta J}{J_u} \Gamma(1 - \alpha) \left(\frac{t}{\tau_2} \right)^\alpha \right] \quad \tau_1 \ll t \ll \tau_2 \quad (15)$$

This model yields a finite initial creep rate

$$\dot{\epsilon} \approx \alpha \delta J \frac{\alpha}{1 - \alpha} \frac{\tau_1^{\alpha-1} - \tau_2^{\alpha-1}}{\tau_2^\alpha - \tau_1^\alpha} \quad (16)$$

A finite creep rate at the origin was achieved differently by Jeffreys [1958], who proposed what he called the 'modified Lomnitz law':

$$\epsilon(t) = \frac{\sigma}{M_u} \left\{ 1 + \frac{q}{\alpha} \left[\left(1 + \frac{t}{\tau} \right)^\alpha - 1 \right] \right\} \quad (17)$$

We note that the time constant in Jeffreys' equation is related to the shortest relaxation time in the spectrum, τ_1 . The parameter q is related to the ratio τ_1/τ_2 , which can be very small. For a dislocation bowing mechanism, τ is proportional to the dislocation length squared. A spread of one order of magnitude in dislocation lengths gives τ_1/τ_2 equal to 10^{-2} .

EFFECTIVE ELASTIC THICKNESS OF THE LITHOSPHERE

We now estimate the thickness of the 'rheological lithosphere.' Clearly, it is a function of lithospheric age, the magnitude of the load, and the duration of loading. At a given depth and therefore temperature the rheology will appear elastic for times shorter than the characteristic relaxation time, and viscous for longer duration loads. The 'thickness of the lithosphere' is therefore a more complicated concept than has been generally appreciated.

In Figure 2 we present estimates of the thickness of the oceanic rheological lithosphere as a function of age of crust and duration of load. In the calculation we have used oceanic geotherms from Schubert *et al.* [1976] and a stress of 1 kbar. The depth to the rheological asthenosphere is defined as the depth having a characteristic time equal to the duration of the load. For example, from Figure 2 a 30 million year old load imposed on 80 million year old crust would yield a thickness of 45 km if the lithosphere did not subsequently cool or if the cooling

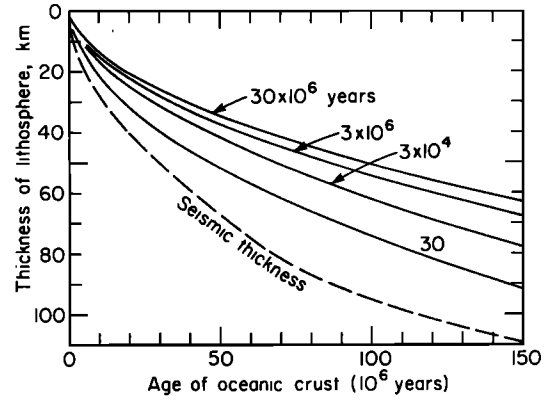


Fig. 2. Relationship between thickness of elastic lithosphere, age of oceanic crust and duration of loading for 1 kbar stress.

time is longer than the relaxation time. A 30 million year old load on currently 80 million year old crust would give a 34-km-thick rheological lithosphere with the above qualifications. The thickness of the high seismic velocity layer overlying the low-velocity zone (the LID) is also shown in Figure 2. For load durations of millions of years the rheological lithosphere is about one half the thickness of the seismological lithosphere. Note that the rheological thickness decreases only gradually for old loads. This is consistent with the observations of Watts *et al.* [1975]. On the other hand, the lithosphere appears very thick for young loads, and the apparent thickness decreases rapidly.

SUMMARY

A solid containing dislocations exhibits nonelastic behavior on a variety of time scales. Stress-induced glide of dislocations is a viable mechanism for seismic wave attenuation in the mantle if it is diffusion-controlled. On the other hand, the steady motion of dislocations is probably responsible for creep in the mantle. Steady state dislocation creep in olivine shows a high activation energy which may point to a contribution of jog nucleation in addition to self-diffusion. If the same population of dislocations contributes to both phenomena, there may be a close connection between attenuation of seismic waves and viscosity. The stresses and time scales associated with these phenomena are, of course, vastly different, and this had led to the opinion that the corresponding physical mechanisms must be unrelated.

In this paper we argue that dislocation motions can plausibly explain both the short-term, low-stress (seismic), and long-term (flow) rheological behavior of the mantle. Under small strains, dislocations bow by glide, and the time scale is controlled by kink or impurity diffusion. At large strains and for long time scales, dislocations can drag their jogs or can climb out of their glide planes, and the motion is controlled by the slower process of self-diffusion.

The pronounced low-velocity zone found in oceanic and tectonic regions can be explained in terms of relaxation by dislocation glide. The time constant appears to be appropriate for explaining the absorption band at seismic frequencies. The higher stress and longer duration rheological properties can be understood in terms of dislocation climb. Both glide and climb are thermally activated processes, and there is therefore a relation between the seismic and rheological properties via the temperature and pressure.

The apparent thickness of the rheological lithosphere depends on temperature, stress, and load duration. For small

loads, for example, postglacial rebound, relaxation times are longer, and lithospheric thicknesses are greater than for large loads such as oceanic islands. For loads older than thousands of years the rheological thickness is much less than the seismic thickness.

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REFERENCES

- Ahrens, T. J., and G. Schubert, Gabbro-eclogite reaction rate and its geophysical significance, *Rev. Geophys. Space Phys.*, **13**, 383-400, 1975.
- Anderson, D. L., and R. S. Hart, Absorption and the low velocity zone, *Nature*, **263**, 397-398, 1976.
- Anderson, D. L., and J. B. Minster, The frequency dependence of Q in the earth and implications for mantle rheology and Chandler wobble, *Geophys. J. Roy. Astron. Soc.*, **58**, 431-440, 1979.
- Ashby, M. F., and R. A. Verrall, Micromechanism of flow and fracture, and their relevance to the rheology of the upper mantle, *Phil. Trans. Roy. Soc. London, Ser. A*, **288**, 59-95, 1978.
- Cara, M., Lateral variations of S -velocity in the upper mantle from higher Rayleigh modes, *Geophys. J. Roy. Astron. Soc.*, **57**, 649-670, 1979.
- Cathes, L. M., III, *Viscosity of the Earth's Mantle*, Princeton University Press, Princeton, 1975.
- Celli, V., M. Kabler, T. Ninomiya, and R. Thomson, Theory of dislocation mobility in semiconductors, *Phys. Rev.*, **131**, 58-72, 1963.
- DeBatist, R., *Internal Friction of Structural Defects in Crystalline Solids*, 477 pp., North-Holland, Amsterdam, 1972.
- Durham, W. B., C. Goetze, and B. Blake, Plastic flow of oriented single crystals of olivine, 2, Observations and interpretations of the dislocation structures, *J. Geophys. Res.*, **82**, 5755-5770, 1977.
- Gittus, J. H., Theoretical equation for steady-state dislocation creep effects of jog-drag and cell formation, *Philos. Mag.*, **34**, 401-411, 1976.
- Goetze, C., The mechanisms of creep in olivine, *Phil. Trans. Roy. Soc. London, Ser. A*, **288**, 99-119, 1978.
- Gueguen, Y., High temperature olivine creep: Evidence for control by edge dislocations, *Geophys. Res. Lett.*, **6**, 357-360, 1979.
- Gueguen, Y., and J. M. Mercier, High attenuation and the low velocity zone, *Phys. Earth Planet. Interiors*, **7**, 39-46, 1973.
- Hirth, J. P., and J. Lothe, *Theory of Dislocations*, 780 pp., McGraw-Hill, New York, 1968.
- Huber, R. J., G. S. Batur, and P. Gibbs, High-temperature kilocycle internal friction in Al_2O_3 single crystals, *J. Appl. Phys.*, **32**, 2573-2579, 1961.
- Ivanov, L. I., and V. A. Yanushkevich, High temperature mechanism of steady state creep in BCC metals: Creep of zirconium, *Fiz. Met. Metalloved.*, **17**, 112, 1964.
- Jackson, D., Grain boundary relaxation and the attenuation of seismic waves, Ph.D. thesis, Mass. Inst. of Technol., Cambridge, 1969.
- Jeffreys, H., Rock creep, *Mon. Not. Roy. Astron. Soc.*, **118**, 14-17, 1958.
- Kohlstedt, D. L., and C. Goetze, Low-stress high-temperature creep in olivine single crystals, *J. Geophys. Res.*, **79**, 2045-2051, 1974.
- Kohlstedt, D. L., C. Goetze, and W. B. Durham, Experimental deformation of single crystal olivine with application to flow in the mantle, in *Petrophysics: The Physics and Chemistry of Minerals and Rocks*, edited by R. G. U. Strens, pp. 35-49, John Wiley, New York, 1976.
- Kumazawa, M., and O. L. Anderson, Elastic modulus, pressure derivatives and temperature derivatives of single-crystal olivine and single-crystal forsterite, *J. Geophys. Res.*, **74**, 5961-5972, 1969.
- Liu, H. P., D. L. Anderson, and H. Kanamori, Velocity dispersion due to anelasticity; implications for seismology and mantle composition, *Geophys. J. Roy. Astron. Soc.*, **47**, 41-58, 1976.
- Macdonald, J. R., Theory and application of a superposition model of internal friction and creep, *J. Appl. Phys.*, **32**, 2385-2398, 1961.
- Menezes, R. A., and W. D. Nix, High temperature dislocation mobility in LiF, II, Charged dislocations in ionic crystals, *Mater. Sci. Eng.*, **16**, 57-66, 1974.
- Miner, R. E., R. Gibala, and F. A. Hultgren, An application of the Schoeck theory to the cold-work internal friction peak in iron, *Acta Metall.*, **24**, 233-239, 1976.
- Minster, J. B., and D. L. Anderson, A model of dislocation-controlled rheology for the mantle, *Phil. Trans. Roy. Soc. London*, in press, 1980.
- Misener, D. V. Cationic diffusion in olivine to 1400°C and 35 kbar, *Carnegie Inst. Washington Publ.*, **634**, 117-129, 1974.
- Nabarro, F. R. N., Steady-state diffusional creep, *Philos. Mag.*, **16**, 231-237, 1967.
- Nowick, A., and B. Berry, *Anelastic Relaxation in Crystalline Solids*, p. 677, Academic, New York, 1972.
- Poirier, J. P., *Plasticité à Haute Température des Solides Cristallins*, Eyrolles, Paris, 1976.
- Poumellec, M., O. Jaoul, C. Froidevaux, and A. Havette, Diffusion in forsterite: A new constraint for understanding mantle deformation, preprint, 1980.
- Reddy, K. P. R., S. M. Oh, L. D. Major, Jr., and A. R. Cooper, Oxygen diffusion in forsterite, *J. Geophys. Res.*, **85**, 322-326, 1980.
- Schoeck, G., Fricción interna debido a la interacción entre dislocaciones y átomos solutos, *Acta Metall.*, **11**, 617-622, 1963.
- Schubert, G., C. Froidevaux, and D. A. Yuen, Oceanic lithosphere and asthenosphere: Thermal and mechanical structure, *J. Geophys. Res.*, **81**, 3525-3540, 1976.
- Simpson, H. M., and A. Sosin, Contribution of defect dragging to dislocation damping, I, Theory, *Phys. Rev., Sect. B*, **5**, 1382-1393, 1972.
- Solomon, S. C., On Q and seismic discrimination, *Geophys. J. Roy. Astron. Soc.*, **31**, 163-177, 1972.
- Stocker, R. L., Influence of oxygen pressure on defect concentrations in olivine with a fixed cationic ratio, *Phys. Earth Planet. Interiors*, **17**, 118-129, 1978.
- Stocker, R. L., and M. F. Ashby, On the rheology of the upper mantle, *Rev. Geophys. Space Phys.*, **11**, 391-497, 1973.
- Watts, A. B., J. R. Cochran, and G. Selzer, Gravity anomalies and flexure of the lithosphere: A three-dimensional study of the Great Meteor Seamount, NE Atlantic, *J. Geophys. Res.*, **80**, 1391-1398, 1975.
- Weertman, J., High temperature creep produced by dislocation motion, paper presented at the John E. Dorn Memorial Symposium, Cleveland, Ohio, Oct. 17, 1972.

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